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## Research Article

# Ant intelligence for solving optimal path-covering problems with multi-objectives 

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#### Abstract

Conventional methods have difficulties in forming optimal paths when raster data are used and multi-objectives are involved. This paper presents a new method of using ant colony optimization (ACO) for solving optimal pathcovering problems on unstructured raster surfaces. The novelty of this proposed ACO includes the incorporation of a couple of distinct features which are not present in classical ACO. A new component, the direction function, is used to represent the 'visibility' in the path exploration. This function is to guide an ant walking toward the final destination more efficiently. Moreover, a utility function is proposed to reflect the multi-objectives in planning applications. Experiments have shown that classical ACO cannot be used to solve this type of path optimization problems. The proposed ACO model can generate near optimal solutions by using hypothetical data in which the optimal solutions are known. This model can also find the near optimal solutions for the real data set with a good convergence rate. It can yield much higher utility values compared with other common conventional models.


Keywords: Ant colony optimization; Path-covering; Multi-objectives; Site selection; GIS

## 1. Introduction

A common type of site selection problems in geographical information system (GIS) involves the identification of a number of points (optimized locations) for sitting facilities, such as factories, schools, hospitals, shopping centers, and warehouses. Heuristic search methods can be developed in GIS to tackle these point-sitting and covering problems (Li and Yeh 2005). Another type of spatial optimization problems is related to line features or path finding. Path-finding problems have attracted widespread research interests in many disciplines, such as robot path planning (Kruusmaa and Willemson 2003), emergency evacuation, logistics management, infrastructure planning, and travel demand analysis (Tanga and Pun-Cheng 2004). The objective is to choose the best travel path according to the costs in terms of time, distance or safety (Kruusmaa and Willemson 2003). The path-covering optimization is usually much more complex than the point-covering optimization. It is because a path consists of many connected cells (points). The optimization involves a huge solution space because there are an infinite number of possible routes between origins and destinations on a continuous surface (Zhang and Armstrong 2008).

[^0]Functions have been provided in most GIS to find the optimal paths between origins and destinations based on the least-cost. This type of approach usually adopts the arc-node network model. The Dijkstra's algorithm, which has been widely used to solve the shortest-path problem, is to find the shortest path from a single source node to all other nodes in a network (Evans and Minieka 1992). It should be noted that these techniques for path optimization are mainly carried out with respect to costs. The connectivity property of node-arc representation of road networks is required with the support of the network data storage structure (Tanga and Pun-Cheng 2004). The Dijkstra's algorithm is designed for tracing the shortest path in a network on an accumulated cost surface with nodes connected by weighted links. A virtual network has to be constructed to use this algorithm in the grid layers of GIS (Yu et al. 2003). Many such studies have been reported on fine-tuning of the least-cost-path algorithms for solving real-world problems (Collischonn and Pilar 2000, Zhu et al. 2001, Yu et al. 2003).

There are a few studies on solving path optimization problems in a raster space. Dijkstra's (1959) shortest path algorithm has been revised to find the least-cost-path in a raster data format (Yu et al. 2003). Their studies indicate that the least-costpath optimization based on raster data can generate more practical solutions than that on vector data. However, the methods of path-covering optimization in a raster space have not been well explored because of the complexities. An example is to identify the optimal path across a region for providing the maximum service coverage to various types of discrete targets (e.g. settlements). In transport planning, maximum population coverage along the path is a major concern as well as the minimum transportation distance. This type of optimization problem is very difficult since the population usually has an inhomogeneous and discrete distribution. The solution space becomes extremely huge if multi-objectives are considered for solving these spatial search problems. Although there are a few studies on some simple path-covering problems, these models only concern the optimal coverage of known line features (Boffey and Narula 1998, Huang et al. 2006). The optimal path is selected among these known networks, which are used as the constraints for the optimization. This situation may not be true for many planning applications. For example, the question may be the creation of an optimal path for building a subway with the maximum service coverage to the population. The use of raster surfaces can provide a more convenient environment for solving this type of optimization problem. However, existing algorithms are not expected to generate such optimal paths on raster surfaces by considering service coverage.

Recently, natural swarm intelligence has been used to tackle a variety of complex computation problems, such as functional optimization, route finding, scheduling, structural optimization, vehicle routing and image and data analysis (Sharma et al. 2006). The swarm intelligence is usually designed by imitating the flocking of birds or the swarming of insects such as bees and ants. Ant colony optimization (ACO) which was first proposed by Dorigo et al. (1991) is to solve various optimization problems by simulating the behavior of ants in seeking foods. In spite of the simplicity of ant individuals, ant colonies present a highly structured social organization for completing complex tasks. Studies indicate that a set of cooperating artificial ants with simple swarm intelligence can effectively solve hard optimization problems (Colorni et al. 1991). For example, ACO was used to solve the traveling salesman problem (TSP) (Dorigo and Gambardella 1997a, b). The program of Ant-Miner is developed to discover patterns from raw data (Parpinelli
et al. 2002). Attempts have been made on using ant algorithms to find the optimal solutions to the network routing (Kwang and Weng 2002), location and allocation (Sharma et al. 2006), sitting of service facilities (Liu et al. 2005, Li et al. in press) and capturing line features (Huang et al. 2006).

Few studies have been reported on the application of ACO for tackling these path-covering optimization problems involving multi-objectives. This paper will present a new method of using ant intelligence for generating optimal paths under continuous surfaces. Compared with general ACO, this path-covering ACO model has incorporated a number of unique features for facilitating the spatial search. The novelty includes the use of a direction function, which provides some 'vision' capabilities for assisting the walking on unstructured raster surfaces by artificial ants. This function is to balance the trade-off between local attractions and global attractions during the path exploration. A utility function is proposed to represent the multi-objectives of a specific planning application. All these features are crucial for solving hard combinatorial path-covering problems.

## 2. ACO for solving path-covering problems

### 2.1 The ant algorithm

ACO is a kind of computation algorithms for solving optimization problems. The optimization is carried out by simulating the behavior of real ants in finding foods through collaboration. Ants release some substance called pheromone when they pass a route. The released pheromone trail will evaporate with the passing of time. Ants have a strong ability of adapting to possible changes in the environment. They can effectively find the new shortest path once the old one is no longer feasible (e.g. there is a new obstacle in the path). At the beginning of food search, an ant randomly selects a path for exploration. A shorter path is deposited with a larger amount of pheromone trail. Ants move along the path on which pheromone trail is plentiful. As a result, a more amount of pheromone is deposited on this path. At the final stage, all the ants will be attracted to the shortest path because of this positive feedback mechanism. Experiments indicate that ACO is effective for finding the optimal solution under complex combinatorial situations (Dorigo et al. 1991).

The mechanism of classical ACO can be explained through the solution to the TSP which is to find a closed tour of minimal length connecting $N$ given cities (Dorigo et al. 1996). In the algorithm, an artificial ant chooses a city to visit with a probability which is determined by two components: (1) the amount of pheromone trail $\tau_{u v}(t)$ on the path; and (2) the visibility (travel distance) $\eta_{u v}(t)$. In addition, a tabu list, $\operatorname{tab} u_{k}$, is used to prevent an ant from going to the visited cities again. This probability that an ant moves from city $u$ to city $v$ is given as follows (Dorigo et al. 1996):

$$
p_{u v}^{k}(t)= \begin{cases}\frac{\left[\tau_{u v}(t)\right]^{\alpha} \cdot\left[\eta_{u v}(t)\right]^{\beta}}{\left.\sum_{k \in S_{k}} \tau_{u k k}(t)\right]^{x} \cdot\left[\eta_{u k}(t)\right]^{\beta}} & \text { if } v \in S^{k}  \tag{1}\\ 0 & \text { otherwise }\end{cases}
$$

where $p_{u v}^{k}(t)$ is the transition probability from city $u$ to city $v$ for the $k$ th ant at time $t$, $\tau_{u v}(t)$ is the amount of pheromone trail on path $(u, v)$, and $\eta_{u v}(t)$ is a heuristic function related to the visibility (distance). The set, $S^{k}$, represents the cities that can be visited next time without any repetition.

The parameters of $\alpha$ and $\beta$ control the relative importance of trail versus visibility (distance). A larger value of $\alpha$ indicates that the trail intensity will have more influences on the probability. In contrast, a larger value of $\beta$ means that there is a greater contribution of the visibility (distance) to the probability.

At each iteration $(t)$, the amount of pheromone trail is updated according to the following equations (Dorigo et al. 1996):

$$
\begin{gather*}
\tau_{u v}(t+1)=(1-\rho) \tau_{u v}(t)+\Delta \tau_{u v}(t)  \tag{2}\\
\Delta \tau_{u v}(t)=\sum_{k=1}^{m} \Delta \tau_{u v}^{k}(t) \tag{3}
\end{gather*}
$$

where $\rho$ is a coefficient such that $(1-\rho)$ represents the evaporation of trail between $t$ and $t+n . \Delta \tau_{u v}^{k}(t)$ is the quantity of trail substance per length unit laid on path $(u, v)$ by the $k$ th ant between time $t$ and $t+\mathrm{n}$.
$\Delta \tau_{u v}^{k}(t)$ is calculated according to the following equation (Dorigo et al. 1996):

$$
\Delta \tau_{u v}^{k}(t)= \begin{cases}\frac{Q}{L_{k}} & \text { if the } k \text { th ant } \operatorname{visits}(u, v)  \tag{4}\\ 0 & \text { otherwise }\end{cases}
$$

where $Q$ is a constant, and $L_{k}$ is the tour length or the total travel cost of the $k$ th ant.
An ant has a higher probability of selecting the shorter route between two cities. The heuristic function $\eta_{u v}(t)$ is defined as the inverse of the distance or the visibility between cities $u$ and $v$ (Dorigo et al. 1996):

$$
\begin{equation*}
\eta_{u v}(t)=\frac{1}{d_{u v}} \tag{5}
\end{equation*}
$$

where $d_{u v}$ is the distance between city $u$ and city $v$.

### 2.2 Modifying ACO for adapting to optimal path-covering problems

2.2.1 The modified ant algorithm. Ant algorithms seem to be straightforward for solving path-finding problems because of ants' strong exploration capability. Artificial ants with simple swarm intelligence could be used to determine the best paths for constructing subways, highways and pipelines. In most situations, the optimization should be implemented on raster surfaces for solving practical problems. A number of objectives, such as the minimum total travel distance and maximum service coverage, could be incorporated for the optimization. The formation of an optimal path under such situations is a hard combinatorial optimization problem.

In the optimization, an artificial ant can visit any cell on unstructured raster surfaces and deposit pheromone on the visited cells during path exploration. The process is based on the positive feedback of ant intelligence. However, optimal path formation is a difficult process because of using unstructured surfaces and multiobjectives. An ant could move randomly on the two-dimensional raster surface if there are no constraints and regulations. Therefore, conventional ACO should be modified for adapting to the path-covering problems by incorporating a number of unique features: (1) a direction function for providing some 'vision' capability; (2) a utility function for representing multi-objectives; and (3) strategies for tabu updating and pheromone updating.


Figure 1. An ant visits one of its eight neighboring cells $\left(v_{i}\right)$ from a central cell $(c)$.

During ants' walking, there are eight direct neighboring cells ( $v_{i}$ ) for a central cell (c) on a raster surface (figure 1). These eight neighbors are represented by $v_{1}, v_{2}, v_{3}$, $v_{4}, v_{5}, v_{6}, v_{7}$ and $v_{8}$ for eight possible moving directions. There are infinite combinations of the walking schemes by forming a path between an origin and a destination. An ant could move randomly without finding optimal paths based on conventional ant algorithms. For better convergence, the traditional heuristic function for visibility is replaced by using a more sophisticated direction function $\left[\xi\left(\theta_{c v_{i}}\right)\right]$. The transition probability is then revised by using the following equation:

$$
p_{c v_{i}}^{k}(t)=- \begin{cases}\frac{\left[\tau_{c v_{i}}(t)\right]^{\alpha} \cdot\left[\xi\left(\theta_{c_{i}}(t)\right)\right]^{\beta}}{\sum_{k \in S^{k}}\left[\tau_{c_{v_{i}}}(t)\right]^{\alpha} \cdot\left[\xi \left[\left(\theta_{c_{i} i}(t)\right]^{\beta}\right.\right.} & \text { if } v_{i} \in S^{k}  \tag{6}\\ 0 & \text { otherwise }\end{cases}
$$

where $c$ is the central cell (the current occupied position) for ant $k, v_{i}$ is one of the eight neighbors (moving direction) that the ant will move to at time $t$, and $S^{k}$ is the cells can be visited next time without any repetition.

Like many heuristic algorithms, ACO uses a roulette-wheel selection mechanism to decide which cells will be visited based on the probability. In the virtual wheel, each moving direction is associated with a sector which has its area proportional to the probability. This assumes that a larger sector will be more likely selected based on the gambling strategy. As a result, an ant will have a greater chance to walk in the direction of a larger probability value. It is apparent that the results from different simulations are not totally the same. The created paths should be subject to some uncertainties because of the stochastic characteristics.

Path optimization usually concerns two common objectives in planning practice: (1) the maximum service coverage to the population (benefits); and (2) the minimum total travel distance (costs). A path that can provide the service to the covered population ( $P_{\text {pathcov }}$ ) within the buffer of $R_{c o v}$ subject to the total travel length $\left(L_{p a t h}\right)$. The optimization should be implemented by satisfying these two objectives as much as possible.
2.2.2 The pheromone component. An ant will move from the central cell to one of its eight direct neighbors at each step of path exploration. The transition probability is partially related to the pheromone density $\left[\tau_{c v_{i}}(t)\right]$ according to equation (6). An ant is expected to walk toward the neighboring cell which has the largest amount of pheromone. However, the difference of pheromone is very small between these eight direct neighbors because of spatial autocorrelation. This could cause the ant to walk in a random direction. Therefore, the pheromone at the cells of a further distance $(r)$ should be used to determine the probability of walking direction correctly (figure 2). For example, the pheromone at $v_{3}$ is replaced by that at $V_{3}^{\prime}$ which has the largest


Figure 2. Ants attracted by the pheromone at a further distance.
amount of pheromone at that direction:

$$
\begin{equation*}
\tau_{c v_{i}}(t)=\tau_{c V_{i}}(t) \tag{7}
\end{equation*}
$$

At the beginning, each cell has an equal amount of pheromone density $\left[\tau_{c v_{i}}(0)=0\right]$. The amount of pheromone trail will increase if the cell is visited by an ant during the optimization. The evaporation will cause the trail density to decrease if the cell is not visited. The amount of pheromone trail is updated according to equations (2) and (3).

The variable, $L_{k}$, in equation (4) represents the tour length or the total travel cost of a path for the $k$ th ant. The term of $1 / L_{k}$ can be regarded as the utility of this path. A more general form for representing this term is based on a utility function, $\mathrm{f}_{k}$. This function provides a convenient means to evaluate the quality of a path because objectives can be conveniently incorporated. For example, this function can be defined by addressing these two conflicting objectives, the maximum service coverage and the minimum travel distance. A way to combine these two conflicting objectives is based on the following ratio function:

$$
\begin{equation*}
\mathrm{f}_{k}=\frac{P_{k}}{L_{k}} \tag{8}
\end{equation*}
$$

where $P_{k}$ is the total covered population along the path within the buffer of $R_{c o v}$, and $L_{k}$ is the tour length for the $k$ th ant respectively.

Equation (4) is thus revised to incorporate this utility function:

$$
\begin{align*}
\Delta \tau_{c v_{i}}^{k}(t) & = \begin{cases}Q \mathrm{f}_{k} & \text { if the } k \text { th ant } \operatorname{visits}\left(c, v_{i}\right) \\
0 & \text { otherwise }\end{cases} \\
& = \begin{cases}\frac{Q P_{k}}{L_{k}} & \text { if the } k \text { th ant } \operatorname{visits}\left(c, v_{i}\right) \\
0 & \text { otherwise }\end{cases} \tag{9}
\end{align*}
$$

The advantage is that this utility function can be conveniently defined and modified for a specific application without changing the model structure. Multiobjectives could be incorporated by modifying this utility function. Although this study only defines two objectives (the maximum service coverage and the minimum travel distance), more objectives can be incorporated by properly defining this utility function. For example, a linear weighted combination could be used to represent more than two objectives. The definitions of this utility function should be subject to domain knowledge. The use of this utility function can thus allow ACO adapted to the solution of various path optimization problems with multi-objectives.
2.2.3 The heuristic component. The traditional heuristic function cannot be directly used for implementing complex path optimization on raster surface. In this study, the heuristic function is replaced by using a direction function, which considers both local attractions and global attractions for providing some 'vision' capability. An ant explores various possible combinations for generating the maximum utility in terms of these two objectives - service coverage and travel distance. For satisfying the first objective, the movement of an ant is guided by the population density in local neighborhood. The ant will be attracted to the cells of large local population density so that the path can cover the maximum population there. It is expected that the total travel distance will become much longer than that of the optimal path if an ant is guided by just considering the factor of local population density. For satisfying the second objective, the ant is encouraged to move towards its final destination during path exploration. Therefore, the movement of an ant is heuristically guided by two 'forces' - towards the cells of large local population coverage and towards the final destination.

The direction function is devised to balance the trade-off between the maximum local service coverage and the minimum total travel distance (toward the final destination). First, the local covered population ( $P_{\text {localcov }}$ ) is obtained by summing the population according to the buffer of $R_{\text {cov }}$ (figure 3). The cell $\left(E_{\max }\right)$ of the maximum local covered population ( $P_{\text {localmax }}$ ) in the neighborhood is identified for representing local attractions. An ant is encouraged to walk in the direction of the maximum local population coverage. However, the ant should also move toward the final destination with some incentives. A direction function is defined to address this trade-off (figure 3):

$$
\begin{equation*}
\xi\left[\theta_{c v_{i}}(t)\right]=\exp \left\{\mu \cos \left[\theta_{1}(t)\right]+v \cos \left[\theta_{2}(t)\right]\right\} \tag{10}
\end{equation*}
$$

where $\theta_{1}$ is the angle between the line $\left(c v_{i}\right)$ and the line $\left(c E_{\text {max }}\right), \theta_{2}$ is the angle between the line $\left(c v_{i}\right)$ and the line $(c D), \mu$ is the weight for the attraction of the service coverage to the population, and $v$ is the weight for the attraction of the destination.


Figure 3. The trade-off between the local population coverage and the total travel distance.
The above direction function plays a role on determining which direction an ant will probably move toward at the next step. The first component, which is in favor of smaller angle $\theta_{1}$, is to address the attraction of the maximum local population coverage. A larger value of $\xi\left(\theta_{c v_{i}}\right)$ will be obtained if the ant moves toward the neighboring cell which has the largest population density. The second component, which is in favor of smaller angle $\theta_{2}$, is to guide the ant toward the ultimate destination.

The value of $v$ is set to one before $\mu$ can be heuristically defined. There are usually substantial variations of $P_{\text {localmax }}$ at different central cells because of the inhomogeneous distribution of population. The optimal path is more likely to pass the neighborhood of a larger value of $P_{\text {localmax }}$ for producing large service coverage. An ant should not spend too much time on exploring the neighborhood which has a small population. The ant will probably move toward the destination if there is no large population in the neighborhood. The weight, $\mu$, is then used to reflect the variation of $P_{\text {cavmax }}$ in the whole region:

$$
\begin{equation*}
\mu=2 P_{\text {localmax }} / P_{\text {localmax }}^{\prime} \tag{11}
\end{equation*}
$$

where $P^{\prime}{ }_{\text {localmax }}$ is the maximum of $P_{\text {localmax }}$ in the study area.
2.2.4 A modified strategy for updating the tabu list. A special strategy is proposed to define the tabu list for facilitating the optimal path formation. In TSP, the tabu list only stores the visited cities which should not be visited again in the next iterations by an ant. This strategy is not completely appropriate for this study because the number of unvisited cells is very large. More cells should be included in the tabu list for two reasons. Firstly, if an ant just explores a neighborhood, the increase in the service coverage cannot compensate the increase in the travel distance. Secondly, unconstrained walking will cause the overlapping of the service coverage and thus reduce the total served population. A special strategy of using a stricter tabu list is proposed to avoid these two situations. This strategy is to encourage an ant to move forward instead of backward according to a coding system. This strategy is different from that of traditional TSP methods.

First, the codes, $i=1,2,3, \ldots$, are given to represent the backward positions of a walking ant from the current cell to the original one (figure4). A larger buffer distance $\left(R_{t a b u}^{i}\right)$ around the path is defined with the increase of $i$ (moving backward). The cells within the buffer are included in the tabu list although they are not visited before. This can significantly reduce the chance of walking backward by an ant. The buffer distance $\left(R_{\text {tabu }}^{i}\right)$ is defined by using the following simple rules:

$$
R_{t a b u}^{1}=1.0 \text { when } i=1 \text { and }
$$



Figure 4. A strategy of using a stricter tabu list for encouraging ants moving forward.
$R_{t a b u}^{i}=i \times \frac{\sqrt{2}}{2}$ when $i>1$ (The buffer distance increases with the increase of $i$ );
if $R_{t a b u}^{i}>2 \times R_{c o v}$ then $R_{t a b u}^{i}=2 \times R_{c o v}$
2.2.5 Additional strategies for pheromone updating. Equations (2), (3) and (9) are the basis of pheromone updating. In traditional TSP, the pheromone is increased if a place is visited by an ant. The path formation cannot be completed by just using this simple strategy. Some modifications are required for producing more effective pheromone updating. First, the pheromone at the cells near the destination (e.g. within a $10 \times 10$ window around the destination) is set to high values so that ants can more easily move towards the destination. Moreover, the following three strategies are used for pheromone updating:

1. A cell may be passed by many possible routes in the exploration. If the pheromone of a cell is updated during each visit, the information of the best route can be lost because of the average effects of all the routes. Therefore, only the best utility value will be recorded for a cell if it has many routes passed. This can prevent the path formation from trapping at local optimum.
2. A moving window of $3 \times 3$ is then used to rank the best utility values in the neighborhood. Only these cells with the first top three utility values will update the pheromone. The purpose of steps $1-2$ is to keep the important paths, but ignoring some trivial paths.
3. A 'minority' strategy is complementarily adopted to maintain the diversity of solutions. This is crucial for preventing the degradation of solution population. Some incentives are given to new emerging paths although they have relative low amounts of pheromone at the beginning. Only these cells will update the pheromone if their pheromone is higher than the average pheromone.
2.2.6 Posterization procedures. A path generated by this proposed algorithm may still be subject to some noises or uncertainties because of the large solution space. Some posterization procedures are required to improve the path shape. A constructed path may consist of some redundant cells or unnecessary curves. In this study, the thinning procedure is based on four templates for removing redundant cells (figure $5(a)$ ). The straightening procedure is based on eight templates for removing unnecessary curves (figure $5(b)$ and $(c)$ ).

## 3. Model implementation and results

Experiments were carried out to test if the proposed ACO model can solve the difficult path optimization problems. Three sets of spatial data were used to facilitate the validation (figure 6). The first two sets are hypothetical data in which the optimal solutions are known. The use of hypothetical data can allow the easy verification of the optimization results. The third set of data is from the 2003 census data of Guangzhou, which is available at the street-block unit. All these data have the same size of $250 \times 250$ cells. In the last set of data, each cell represents an area of $100 \times 250 \mathrm{~m}$ on the ground.

Table 1 lists the parameters that are used in this proposed ACO model. The first five parameters were set according to classical ACO (Dorigo et al. 1996). $R_{c o v}$ was


Figure 5. Posterization procedures of thinning and straightening for improving the final shape of a path: (a) thinning; (b) straightening $1 ;(c)$ straightening 2.


Figure 6. Three types of population density surfaces: (a) hypothetical data set 1 ; (b) hypothetical data set $2 ;$ (c) census data of Guangzhou.

Table 1. Parameters used in this ACO path-covering model.

| Iteration | Ants | $\alpha$ | $\beta$ | $\rho$ | $R_{\text {cov }}$ | $r$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 200 | 20 | 2 | 1 | 0.9 | 10 | 3 |

specifically used to represent the buffer distance that can provide the service to the population. In this study, the value of $R_{\text {cov }}$ was assumed to be 10 , which is 1 km on the ground. The pheromone at the cells of a further distance $(r)$ was used to determine the probability of walking direction correctly. The appropriate value of $r$ was determined by experiments. Figure 7 has clearly indicated that the best utility value can be obtained when $r=3$ for the study area.

In the first scenario, the population density decreases from the ridge (central line), and the origin and the destination are just situated on the ridge (figure 6(a)). It is easy to know that the optimal path should follow the central line for generating the maximum utility. Figure $8(a)$ and table 2 clearly show that the proposed ACO can find the near optimal solution because their shapes and utility values are very close


Figure 7. Determining the best value of $r$ by experiments.


Figure 8. Near optimal paths identified by the proposed ACO for the hypothetical data sets: (a) hypothetical data set $1 ;(b)$ hypothetical data set 2 .

Table 2. The utility values between the known result and the simulation for scenario 1 .

| Known result | ACO | The difference |
| :--- | :---: | :---: |
| 11509 | 11407 | $0.89 \%$ |

between the known result and the simulation. The difference is only $0.89 \%$ in terms of their utility values.

In the second scenario, the destination is located away from the ridge (figure $6(b)$ ). This pattern is slightly more complicated compared with that of the first scenario (figure $6(a)$ ). Figure $8(b)$ shows the optimal path identified by this proposed ACO method. The optimal path follows the central line for the first half part, and then switches to the destination for the second half part.

This ant algorithm was then applied to a real data set for testing its effectiveness of solving practical spatial optimization problems. The original vector data were converted into raster data with the resolution of 100 m . The actual population distribution is much more complex than those of hypothetical data (figure 6(c)). Figure 9 is the simulation result for the near optimal path using ant intelligence. Figure 10 shows that the average utility value and the maximum utility value of these paths will both increase significantly at the initial stage. These values will become stabilized when the iteration reaches about 100-120. This indicates that this ant algorithm has a good convergence rate for finding the optimal path or near optimal one. One simulation will spend about 4 hours for completing an optimal path by using a computer with a Pentium IV 3.2 GHz CPU.


Figure 9. A near optimal path identified by the proposed ACO for Guangzhou data set.

Average utility

(a)

Maximum utility

(b)

Figure 10. Utility improvement with iterations by the proposed ACO: (a) average utility; (b) maximum utility.

Figure 11 clearly shows the advantages of this proposed ACO over classical ACO for the path-covering optimization. The process of path exportation is based on the pheromone updating of artificial ants. Figure 11(a) shows the optimization process by using this proposed ACO which has incorporated more heuristics. At the beginning, all the cells have equal amount of pheromone. Initial potential paths will

Proposed ACO
Iterations=0 Iterations=50 Iterations=100
Pheromone

(a)

(b)

Figure 11. Optimal path formation based on pheromone updating: (a) proposed ACO; (b) classical ACO.
appear if some cells are visited more often. Some of these paths will be gradually deposited with more amounts of pheromone if they can generate higher utility values. The pheromone on the background cells will evaporate because they are less visited by ants. At the final stage, the optimal path is identified as other paths disappear according to the positive feedback mechanism.

Figure 11(b) shows the process of optimal path exploration by using the classical ACO model. It is very clear that the pheromone will disappear at the midway from the origin to the destination. An ant cannot complete a path because of the lack of the vision capability. Therefore, the classical ACO is unsuitable for solving these path-covering problems. The used of the direction function is crucial for guiding an ant towards the final destination.

Heuristic algorithms are subject to some uncertainties. The stability of repeated simulations is another important indicator for assessing the validity of the proposed model. Figure $12(a)$ is the overlay of the optimal paths from 10 repeated simulations. It clearly shows that the proposed ACO can repeat the simulation results although there are some minor differences. Moreover, the simulations can be also repeated by changing the walking direction. There are two different types of walking - forward (from the origin to the destination) and backward (from the destination to the origin). It is expected that the simulated patterns should be very similar between the forward walking and backward walking. This assumption has been confirmed by the experiment (figure $12(b)$ ). Table 3 also compares the best utility values obtained from 10 different simulations by the forward walking and backward walking respectively. The variations are very small because these two types of walking yield the similar mean values and small standard deviation values.


Figure 12. The stability of repeated simulations between (a) forward and (b) backward walking.

Table 3. The best utility values obtained from 10 different simulations.

|  | $\operatorname{Sim} 1$ | $\operatorname{Sim} 2$ | $\operatorname{Sim} 3$ | $\operatorname{Sim} 4$ | $\operatorname{Sim} 5$ | $\operatorname{Sim} 6$ | $\operatorname{Sim} 7$ | $\operatorname{Sim} 8$ | $\operatorname{Sim} 9$ | $\operatorname{Sim} 10$ | Mean | Std. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Forward <br> walking | 8479 | 8245 | 8626 | 8694 | 8701 | 8578 | 8214 | 8403 | 8312 | 8639 | 8489.1 | 185.6 |
| Backward <br> walking | 8629 | 8390 | 8304 | 8365 | 8599 | 8727 | 8534 | 8237 | 8443 | 8514 | 8474.2 | 154.1 |

The evaluation of this proposed model can also be performed by comparing it with some conventional methods. Since the utility function as described in equation (8) represents the two objectives (the maximum service coverage and the minimum travel distance), it is fair to use this function as an indicator for the comparison. A problem is that existing methods cannot completely address these two objectives simultaneously. We have to use two available methods, modified Dijkstra's algorithms, for the comparison. The first experiment is based on the modified Dijkstra's least-cost algorithm using raster data (Yu et al. 2003). It can generate the least accumulated cost, but cannot generate the maximum population coverage around the path. Therefore, it cannot produce the highest utility value. The second experiment is to use a moving window of $10 \times 10$ to sum the population first. The Dijkstra's least-cost algorithm is then applied to this summed population surface for generating the path. This method has some buffering effect for addressing the population coverage consideration.

Figure 13 is the paths formed by these two modified Dijkstra's algorithms. Figure 14 compares the total utility values of these paths generated by these two


Figure 13. The paths identified by the Dijkstra method: (a) Dijkstra; (b) Dijkstra-buffer.


Figure 14. Comparison of the total utility values between the modified Dijkstra methods and the proposed ACO method.

Dijkstra's methods and the proposed ACO method. It is obvious that the simple Dijkstra path has the least utility value. The Dijkstra-buffer path shows slight improvement of the utility over the simple Dijkstra path. However, it is only the proposed method that can produce the highest utility value among all these methods. The proposed method has the improvement of the utility value by 28.3 and $23.1 \%$ respectively, compared with the simple Dijkstra and the Dijkstra-buffer methods.

## 4. Conclusion

Path optimization often needs to satisfy both maximum service coverage (benefits) and minimum development costs. The incorporation of artificial intelligence in GIS is crucial for solving this type of multi-objective optimization problems. Conventional path-finding models are mainly based on the network data model. Spatial search could become much more difficult to solve if it is implemented on unstructured raster surfaces. In some situations, optimal paths should be created from scratch without prior knowledge of networks. Although there are some studies on networking analysis, path-covering problems have received much less attention compared with the point-covering problems. It is because the path-covering problems with multi-objectives on unstructured raster surfaces have a huge solution space. Even conventional heuristics have difficulties in finding the feasible solutions to these hard combinatorial problems.

This study has demonstrated that simple ant intelligence can provide a promising tool for solving optimal path-covering problems with multi-objectives. Artificial ants are faced with a huge set of combinatorial options during path exploration. Significant modifications should be carried out for adapting ACO to the solution of path-covering problems. Experiments have shown that classical ACO cannot complete a path on raster surfaces because of the lack of vision capability. Artificial ants cannot be guided toward the destination because the pheromone will completely disappear away from the origin. This type of algorithm is inappropriate for solving the multi-objective path-covering problems. Instead, the proposed ACO has strong capability for path optimization by incorporating a number of strategies (e.g. the use of direction function and the modifications of pheromone updating). The use of the utility function is crucial for adapting ACO to path optimization. This utility function could be conveniently modified according to domain knowledge without altering the structures of the ant algorithm. This flexibility is desirable as it enables the model suitable for various applications.

The proposed model has been tested by using hypothetical and real data sets. There is a question if this proposed ACO model will produce a result converged towards a local optimum. In most situations, the verification is very difficult because the optimum is unknown. Some hypothetical data with typical population distribution can be used to examine the optimization effects. In this study, the first two data sets have known results because of the typical population distribution. The comparison indicates a strong concordance between the simulation results and the known results. The difference is only $0.89 \%$ in terms of the utility value.

Although the population surface is quite complex for real data, the greedy heuristic of distributed ants yields a good convergence rate (stabilized after 150-200 iterations) in the optimal path search. Moreover, the proposed method can produce quite stable results from repeated simulations. There are also no significant changes of optimization results if the movement direction is reversed from forward walking
to backward walking. The standard deviations are very small for these repeated simulations.

Comparisons have been carried out between this proposed ACO method and conventional methods. However, conventional methods are not straightforward for the optimization on the raster surfaces. Two modified Dijkstra methods, the simple Dijkstra and the Dijkstra-buffer, are used for the comparison. The analyses indicate that the proposed ACO method can yield higher utility values than the simple Dijkstra and the Dijkstra-buffer methods by 28.3 and $23.1 \%$ respectively. It is because the Dijkstra methods have difficulties in dealing with the multi-objective issue on raster space.

This study only uses two common objectives, the maximum service coverage and the minimum travel distance, for defining the utility function. Future studies should focus on using more than two objectives since numerous constraints and spatial variables should be considered for solving real-world problems. A linear weighted combination could be used to define the utility function so that more than two objectives can be incorporated.

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